9/17 Recitation

1. Let $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix}$. Determine $N(A)$ by finding vectors that span $N(A)$. What is $N(A)$, geometrically? Where does it live? Feel free to use Julia for row reduction.

2. Let $A = \begin{bmatrix} 2 & -1 \\ -1 & 0 \\ 0 & 1 \end{bmatrix}$. Consider the question: Is the vector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ in $C(A)$? Let's solve this in two ways.
   (a) Solve using the interactive graph at `https://www.geogebra.org/m/btwdyx3f`.
   (b) Solve by solving a related system of equations. Feel free to use Julia.

3. (a) Show that if an $m \times n$ matrix A has a pivot in every row, then $C(A) = \mathbb{R}^m$.

(b) In class, we argued geometrically that if $\mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are not scalar multiples of each other, then $\text{Span}\{\mathbf{v}, \mathbf{w}\} = \mathbb{R}^2$ (i.e. the span is a plane). Prove this algebraically by thinking about pivots and using part (a).

(c) **Looking Ahead**: The vectors $\mathbf{v}$ and $\mathbf{w}$ in part (b) are called **linearly independent**. We will see later this week that two $\mathbf{v}$ and $\mathbf{w}$ are linearly independent if and only if the matrix $\begin{bmatrix} \mathbf{v} & \mathbf{w} \end{bmatrix}$ has a pivot in each column. Redo part (b), using this fact.